Cambridge Senior Mathematical Methods AC/VCE Units 1&2
Chapter 3 Quadratics: **Skillsheet 3C**

1 Solve each of the following for x:

a $(2x - 3)(x + 1) = 0$

b $x(5x - 1) = 0$

c $3x(1 - x) = 0$

d $x^2 = 9x$

2 Solve each of the following for x:

a $x^2 + 3x - 10 = 0$

b $x^2 - 8x + 15 = 0$

c $x^2 - 12x + 27 = 0$

d $x^2 + 2x - 99 = 0$

e $x^2 - 4x - 12 = 0$

f $x^2 - 2x - 15 = 0$

3 Solve each of the following for x:

a $2x^2 + 13x - 99 = 0$

b $4x^2 - 4x - 3 = 0$

c $2x^2 + 7x + 3 = 0$

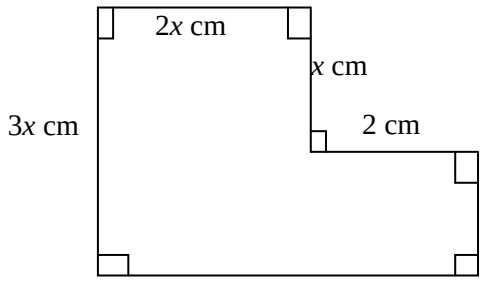
d $6x^2 + 3x - 3 = 0$

e $6x^2 - 13x + 5 = 0$

f $x^2 + 5 = 6x$

g $x^2 + 4x = 21$

4 The area of the shape below is 42 cm^2. Find the value of x.



5 A rectangle has perimeter 40 cm and the area is 51 cm^2. Find the dimensions of the rectangle.

6 A group of people book out a function centre for $1500. Five people withdraw from the group. The remaining people have to pay $25 more each to cover the cost. How many people were in the original group?

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Answers

1 **a** $\frac{3}{2}$ or -1

b 0 or $\frac{1}{5}$

c 0 or 1

d 0 or 9

2 **a** -5 or 2 **b** 5 or 3 **c** 9 or 3

d -11 or 9 **e** 6 or -2 **f** 5 or -3

3 **a** $\frac{9}{2}$ or -11 **b** $-\frac{1}{2}$ or $\frac{3}{2}$ **c** $-\frac{1}{2}$ or -3

d $\frac{1}{2}$ or -1 **e** $\frac{1}{2}$ or $\frac{5}{3}$ **f** 5 or 1

g -7 or 3

4 $\frac{7}{3}$

5 17, 3

6 20 people in the group

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1 Expand each of the following:

a $(x-3)^2$

b $(x+3)^2$

c $(-x+4)^2$

d $\left(x-\frac{3}{4}\right)^2$

2 Factorise each of the following:

a x^2-6x+9

b $x^2-10x+25$

c $x^2-3x+\frac{9}{4}$

d $x^2-5x+\frac{25}{4}$

e $9x^2-66x+121$

3 Solve each of the following by completing the square:

a $x^2+4x-11=0$

b $x^2+8x-25=0$

c $x^2+8x+2=0$

d $2x^2-8x+4=0$

e $2x^2-9x+8=0$

4 Express each of the following in the form $y=a(x-h)^2+k$. Hence state the coordinates of the vertex:

a $y=x^2+2x+8$

b $y=x^2+2x-8$

c $y=x^2+4x-12$

d $y=x^2-2x+10$

e $y=x^2+x$

f $y=x^2-x$

g $y=x^2+3x+9$

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Answers

1 **a** $x^2 - 6x + 9$ **b** $x^2 + 6x + 9$

c $x^2 - 8x + 16$ **d** $x^2 - \frac{3}{2}x + \frac{9}{16}$

2 **a** $(x - 3)^2$ **b** $(x - 5)^2$

c $\left(x - \frac{3}{2}\right)^2$ **d** $\left(x - \frac{5}{2}\right)^2$

e $(3x - 11)^2$

3 **a** $-2 - \sqrt{15}$ or $-2 + \sqrt{15}$

b $-4 - \sqrt{41}$ or $-4 + \sqrt{41}$

c $-4 - \sqrt{14}$ or $-4 + \sqrt{14}$

d $2 - \sqrt{2}$ or $2 + \sqrt{2}$

e $\frac{9 + \sqrt{17}}{4}$ or $\frac{9 - \sqrt{17}}{4}$

4 **a** $y = (x + 1)^2 + 7$; $(-1, 7)$

b $y = (x + 1)^2 - 9$; $(-1, -9)$

c $y = (x + 2)^2 - 16$; $(-2, -16)$

d $y = (x - 1)^2 + 9$; $(1, 9)$

e $y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$; $(-\frac{1}{2}, -\frac{1}{4})$

f $y = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$; $(\frac{1}{2}, -\frac{1}{4})$

g $y = \left(x + \frac{3}{2}\right)^2 + \frac{27}{4}$; $(-\frac{3}{2}, \frac{27}{4})$

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Cambridge Senior Mathematical Methods AC/VCE Units 1 & 2
Chapter 3 Quadratics: **Skillsheet 3G**

1 Solve each of the following inequalities:

a $(x+3)(x-2) \geq 0$

b $(x+5)(x-1) < 0$

c $(2x+3)(x-4) \leq 0$

d $(x+1)(2x-1) > 0$

e $(3-x)(x+2) < 0$

f $(x-4)(3x+1) \geq 0$

g $(2x+3)(3x-2) < 0$

h $(2x+1)(3-4x) \leq 0$

2 Solve each of the following inequalities:

a $x^2 - 2x - 8 \geq 0$

b $x^2 + 5x - 24 < 0$

c $x^2 - 4x + 3 \leq 0$

d $2x^2 + 3x - 9 > 0$

e $-x^2 - 5x + 6 \geq 0$

f $-x^2 - 7x - 12 \geq 0$

g $6x^2 + x > 12$

h $10x^2 + 7x \leq -1$

3 Use a Graphics Display Calculator to solve each of the following inequalities:

a $x^2 - 3x - 5 \leq 0$

b $x^2 + x - 4 > 0$

c $x^2 - 4x + 1 \geq 0$

d $-3x^2 + 7x \leq -8$

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Answers

1 **a** $\{x : x \leq -3\} \cup \{x : x \geq 2\}$

b $\{x : -5 < x < 1\}$

c $\{x : -\frac{3}{2} \leq x \leq 4\}$

d $\{x : x < -1\} \cup \{x : x > \frac{1}{2}\}$

e $\{x : x \leq -2\} \cup \{x : x \geq 3\}$

f $\{x : x < -\frac{1}{3}\} \cup \{x : x > 4\}$

g $\{x : \frac{2}{3} < x < \frac{3}{2}\}$

h $\{x : x < -\frac{1}{2}\} \cup \{x : x > \frac{3}{4}\}$

2 **a** $\{x : x \leq -2\} \cup \{x : x \geq 4\}$

b $\{x : -8 < x \leq 3\}$

c $\{x : 1 \leq x \leq 3\}$

d $\{x : x < -3\} \cup \{x : x > \frac{3}{2}\}$

e $\{x : -5 \leq x \leq 1\}$

f $\{x : -4 < x < 3\}$

g $\{x : x < -\frac{3}{2}\} \cup \{x : x > \frac{4}{3}\}$

h $\{x : -\frac{1}{2} < x < -\frac{1}{5}\}$

3 **a** $\{x : -1.19 \leq x \leq 4.19\}$

b $\{x : x < -2.56\} \cup \{x : x > 1.56\}$

c $\{x : x < 0.268\} \cup \{x : x > 3.732\}$

d $\{x : x < -0.84\} \cup \{x : x > 3.17\}$

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Cambridge Senior Mathematical Methods AC/VCE Units 1 & 2
Chapter 3 Quadratics: **Skillsheet 3I**

1 Determine the discriminant of each of the following quadratics:

a $x^2 + 3x - 3$

b $x^2 + 6x + 9$

c $x^2 - 3x + 4$

d $2x^2 + 5x - 3$

e $3x^2 + 2x + 1$

f $-3 - 6x - 3x^2$

2 Without sketching the graphs of the following quadratics, determine whether they cross or touch the x-axis:

a $y = x^2 - 5x + 3$

b $y = -2x^2 + x - 1$

c $y = x^2 + 10x + 25$

d $y = -4 + 4x - x^2$

e $y = 3x^2 - x + 5$

f $y = -x^2 - x + 2$

3 By examining the discriminant, find the number of roots of:

a $x^2 - 8x + 5 = 0$

b $7x^2 - 2x + 1 = 0$

c $12x^2 - 2x - 7 = 0$

d $x^2 + 2\sqrt{3}x + 3 = 0$

e $-2x^2 - 7x - 8 = 0$

f $2x^2 + 6x + 3 = 0$

4 By examining the discriminant, state the nature and number of roots for each of the following:

a $x^2 - 4x + 12 = 0$

b $x^2 - 3x - 7 = 0$

c $-x^2 - x + 4 = 0$

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d $3x^2 - 10x + 3 = 0$

e $-3x^2 - 2x - 2 = 0$

f $8x^2 + 16x + 8 = 0$

Answers

1 **a** 21 **b** 0 **c** –7 **d** 49 **e** –8 **f** 0

2 **a** Crosses

b Neither

c Touches

d Touches

e Neither

f Crosses

3 **a** Two

b None

c Two

d One

e None

f Two

4 **a** None

b Two Irrational

c Two Irrational

d Two Rational

e None

f One Rational

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1 A family of parabolas have rules of the form $y = ax^2$. For the parabola in this family that passes through the point (2, 12). Find the value of a.

2 A family of parabolas have rules of the form $y = ax^2 + c$. For the parabola in this family that passes through the points $(-1, 4)$ and (0, 8), find the values of a and c.

3 A family of parabolas have rules of the form $y = ax^2 + bx$. For the parabola in this family that passes through the point $(1, -6)$ and has one of its x-axis intercepts at 4, find the values of a and b.

4 A quadratic rule for a particular parabola is of the form $y = a(x-b)^2 + c$. The parabola has vertex (–1, 4) and passes through the point with coordinates (1, –4). Find the values of a, b and c.

5 A parabola has vertex with coordinates $(-1, -3)$ and passes through the point with coordinates (2, 15). Find the equation for the parabola.

6 A parabola has x-axis intercepts 3 and –2 and passes through the point (2, –4). Find the equation of the parabola.

7 A parabola has vertex with coordinates $(-2, 3)$ and y-axis intercept 4. Find the equation for the parabola.

8 A parabola has vertex with coordinates $(2, 5)$ and passes through the point with coordinates (1, 4). Find the equation for the parabola.

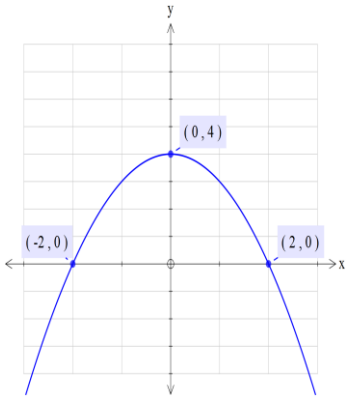
9 A parabola has x-axis intercepts 5 and –1 and passes through the point (–2, 14). Find the equation of the parabola.

10 A parabola has vertex with coordinates $(3, -4)$ and y-axis intercept 1. Find the equation for the parabola.

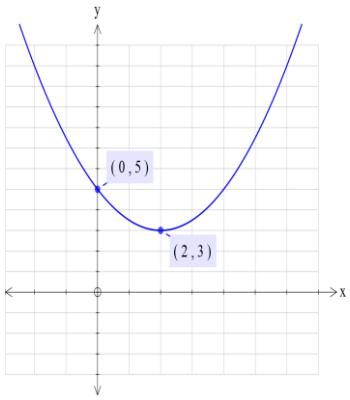
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11 Determine the equation of each of the following parabolas:

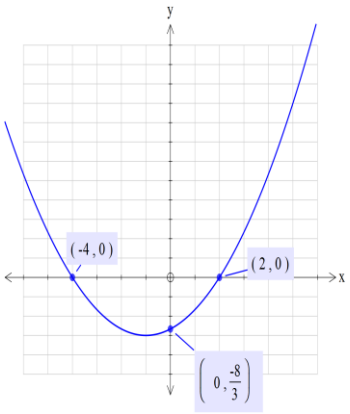
a



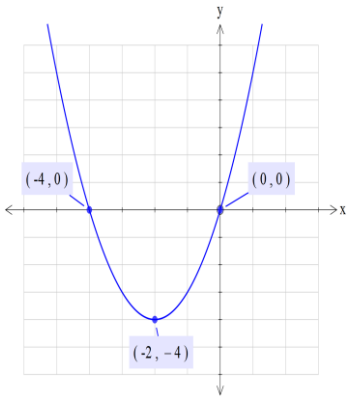
b



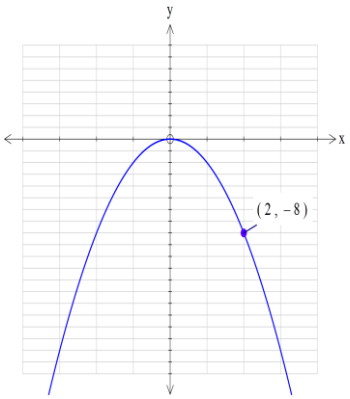
c



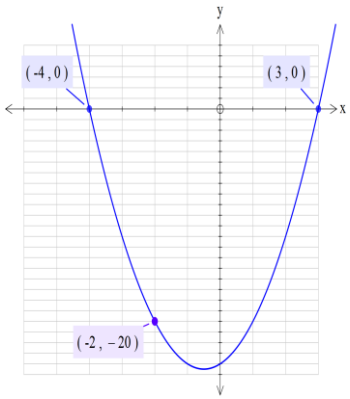
d



e



f



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Answers

1 $y = 3x^2$

2 $y = -4x^2 + 8$

3 $y = 2x^2 - 8x$

4 $y = -2(x+1)^2 + 4$

5 $y = 2(x+1)^2 - 3$

6 $y = (x-3)(x+2)$

7 $y = \frac{1}{4}(x+2)^2 + 3$

8 $y = -(x-2)^2 + 5$

9 $y = (x-5)(x+1)$

10 $y = \frac{5}{9}(x-3)^2 - 4$

11 **a** $y = -x^2 + 4$ **b** $y = \frac{1}{2}(x-2)^2 + 3$

c $y = \frac{1}{3}(x-2)(x+4)$ **d** $y = x(x+4)$

e $y = -2x^2$ **f** $y = 2(x-3)(x+4)$

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1 A piece of wire of length 14 cm is to be formed into the shape of a rectangle. The length of the rectangle is x cm.

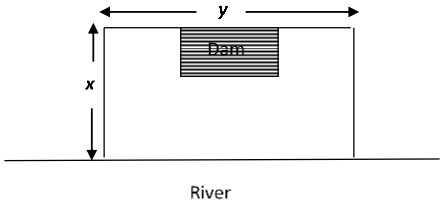
a Find the width of the rectangle in terms of x.

b Find the area of the rectangle, A, in terms of x.

c Sketch the graph of A versus x

d Find the maximum area of the rectangle and the value of x for which the area is a maximum.

2 A farmer uses 1000 metres of fencing to construct a rectangular shaped field. His land is located alongside a river, so he only needs to fence three sides of the field. There is also a rectangular shaped dam of area 5000 m^2 within the field.



Let x be the width of the two shorter sides of the field.

a If y is the length of the field, express y in terms of x.

Let A m^2 be the area of the farmable land inside the fence.

b Show that $A = -2x^2 + 1000x - 5000$

c Sketch the graph of $A = -2x^2 + 1000x - 5000$.

d Use your calculator to find the coordinates of the turning point of the graph.

e Find the dimensions of the field which gives the maximum farmable area, and state the maximum area?

f Either by completing the square or otherwise, express the rule for A in the form $A = a(x-h)^2 + k$

3. The number of tables N sold by an outdoor furniture store during a particular year can be approximated by the equation: $N = m^2 - 14m + 71$, where m is the number of months into the year, $m \in [1,12]$.

a Sketch the graph of the sales for the store over 12 months

b How many tables did the store sell in March?

c In which months did the store sell 47 tables?

d What is the minimum number of tables sold in a month and in which month did this happen?

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The store needs to sell at least 24 tables in a month in order to make a profit.

e For how many months of the year did the store make a profit?

4 A ball is thrown from a balcony that is 20 metres above the ground. The path of the ball is described by the equation: $h = a(x-5)^2 + 22.5$, where *h* metres, is the height of the ball above ground level, when the ball has travelled *x* metres horizontally.

a Show that $a = -0.1$.

b Sketch the graph of $h = -0.1(x-5)^2 + 22.5$.

c What is the maximum height reached by the ball and how far has it travelled horizontally when it reaches its maximum height?

d What is the height of the ball when the horizontal distance from the balcony is 10.5 metres? Give your answer correct to the nearest centimetre.

e What is the horizontal distance of the soccer ball from the balcony when it is 13 metres high?

f How far does it travel horizontally before it hits the ground?

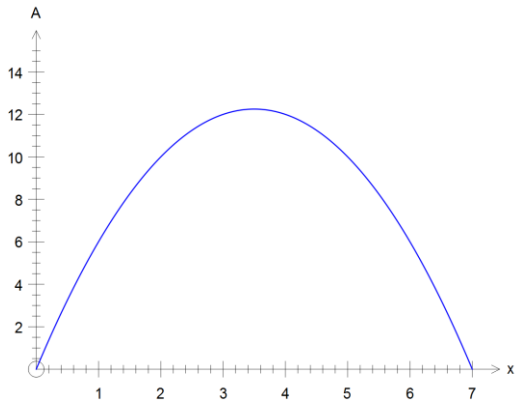
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Answers

1 **a** Width = $7 - x$

b $Area = x(x-7)$

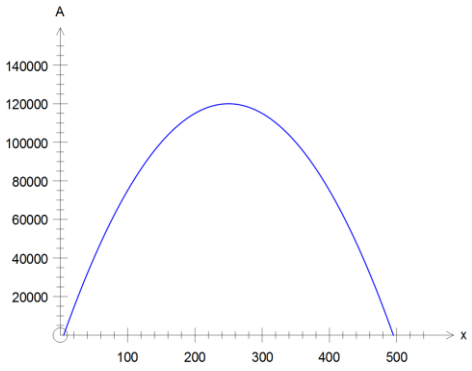
c



d Maximum area is 12.25 cm^2, when x = 3.5cm

2 **a** $y = 1000 - 2x$

c



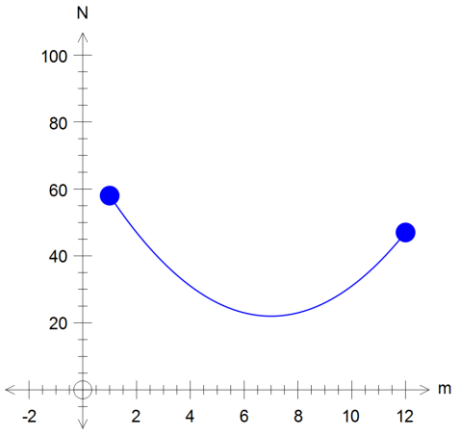
d (250, 12000)

e Field is 25 m × 500 m and the maximum farmable area is 120000 m^2

f $A = -2(x-250)^2 + 120000$

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3 **a**



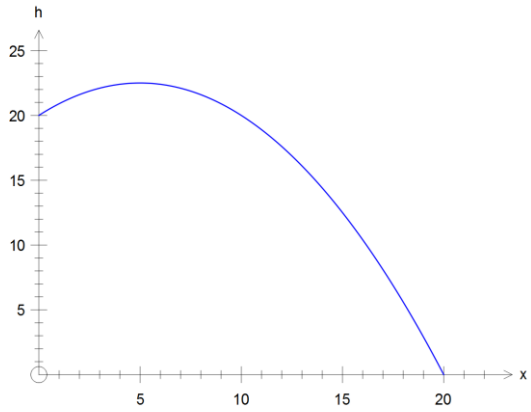
b 38

c February and December

d 22 in July

e 9 months

4 **b** Sketch the graph of $h = -0.1(x-5)^2 + 22.5$.



c 22.5 *m*, 5 *m*

d 19.48 *m*

e 14.75 *m*

f 20 *m*